A Variable Péclet Upstream Weighting Scheme

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The Dead-Oil model

The coupled system

$$\left\{\begin{array}{l} \phi\dfrac{\partial s}{\partial t}+\mathrm{div}\left(\Upsilon(\phi)\ \eta_o(s)\left(\rho_o\overrightarrow{g}-\overrightarrow{\nabla}(p+p_c(s))\right)\right)=0 \text{ on } \Omega\times(0,T) \\ -\phi\dfrac{\partial s}{\partial t}+\mathrm{div}\left(\Upsilon(\phi)\ \eta_w(s)(\rho_w\overrightarrow{g}-\overrightarrow{\nabla}p)\right)=0 \text{ on } \Omega\times(0,T) \end{array}\right.$$

Boundary conditions

$$\left\{\begin{array}{l} \Upsilon(\phi)\ \eta_w(s)\left(\rho_w\overrightarrow{g}-\overrightarrow{\nabla}p\right).\overrightarrow{n}=0 \text{ on } \partial\Omega\times(0,T) \\ \Upsilon(\phi)\ \eta_o(s)\left(\rho_o\overrightarrow{g}-\overrightarrow{\nabla}(p+p_c(s))\right).\overrightarrow{n}=0 \text{ on } \partial\Omega\times(0,T) \end{array}\right.$$

Initial condition

$$s(x,0)=s_{ini}(x)$$

The decoupled system

$$\left\{\begin{array}{l} \mathrm{div}(\overrightarrow{Q}) = 0, \text{ on } \Omega \times (0,T), \\ \phi \dfrac{\partial s}{\partial t} + \mathrm{div}\Big(f(s,\overrightarrow{Q},\overrightarrow{G}) - \Upsilon(\phi)\overrightarrow{\nabla}\varphi(s)\Big) = 0, \text{ on } \Omega \times (0,T), \end{array}\right.$$

where

- $\overrightarrow{Q} = \Upsilon(\phi)\left(\Big(\eta_o(s)\rho_o + \eta_w(s)\rho_w\Big)\overrightarrow{g} - \eta_T(s)\overrightarrow{\nabla}\bar{p}\right)$,
- $\bar{p} = p + \displaystyle\int_0^s \frac{\eta_o}{\eta_T}(v)p_c^{'}(v)dv$,
- $f(s,\overrightarrow{Q},\overrightarrow{G}) = \dfrac{\eta_o}{\eta_T}(s)\overrightarrow{Q} + \dfrac{\eta_o\eta_w}{\eta_T}(s)\overrightarrow{G}$,
- $\overrightarrow{G} = \Upsilon(\phi)(\rho_o - \rho_w)\overrightarrow{g}$,
- $\varphi^{'}(s) = \dfrac{\eta_o(s)\eta_w(s)}{\eta_o(s)+\eta_w(s)}p_c^{'}(s)$.

The finite volume scheme (1)

Discretization of the total mass equation

$$\sum_{L \in N(K)} Q_{K,L}^{n+1} = 0$$

where

- $Q_{K,L}^{n+1} = \Upsilon_{K|L} \left(\left(\eta_{o,K|L}^{n} \rho_o + \eta_{w,K|L}^{n} \rho_w \right) g \delta z_{K,L} - \eta_{T,K|L}^{n} \delta p_{K,L}^{n+1} \right),$
- for all $\alpha \in \{o, w, T\}$

$$\eta_{\alpha,K|L}^{n} = \frac{\Big(d(x_L, K|L) + d(x_K, K|L) \Big) \eta_\alpha(s_K^n) \eta_\alpha(s_L^n)}{\eta_\alpha(s_K^n) d(x_L, K|L) + \eta_\alpha(s_L^n) d(x_K, K|L)}.$$

The finite volume scheme (2)

Discretization of the oil mass equation (upwind scheme)

$$m(K)\phi_K \frac{s_K^{n+1} - s_K^n}{\delta t} + \sum_{L \in N(K)} F(s_K^n, s_L^n, Q_{K,L}^{n+1}, G_{K,L}) - \Upsilon_{K|L}(\varphi(s_L^n) - \varphi(s_K^n)) = 0$$

where

- $G_{K,L} = \Upsilon_{K|L}(\rho_o - \rho_w) g \delta z_{K,L}$,
- $F(s_K^n, s_L^n, Q_{K,L}^{n+1}, G_{K,L}) = \dfrac{(\eta_o)_{K|L}^{n+1}\Big(Q_{K,L}^{n+1} + (\eta_w)_{K|L}^{n+1} G_{K,L}\Big)}{(\eta_o)_{K|L}^{n+1} + (\eta_w)_{K|L}^{n+1}}$,
- the upwind saturations in the terms $(\eta_\alpha)_{K|L}^{n+1}$, $\alpha \in \{o, w\}$ are determined by the values of $Q_{K|L}^{n+1}$ and $G_{K,L}$.

The finite volume scheme (3)

Discretization of the oil mass equation (variable Péclet scheme)

$$m(K)\phi_K \frac{s_K^{n+1} - s_K^n}{\delta t} + \sum_{L \in N(K)} \mathcal{F}(\theta_{K|L}^{n+1}, s_K^n, s_L^n, Q_{K,L}^{n+1}, G_{K,L}) - \Upsilon_{K|L}(\varphi(s_L^n) - \varphi(s_K^n)) = 0$$

where

- $\mathcal{F}(\theta, a, b, Q, G) = \theta F(a, b, Q, G) + (1-\theta) F(\frac{a+b}{2}, \frac{a+b}{2}, Q, G),$
-
$$\theta_{K|L}^{n+1} = \max\left(0, 1 - \frac{\Upsilon_{K|L}(\varphi(s_L^n) - \varphi(s_K^n))}{\Lambda_{K,L}^{n+1}(s_K^n, s_L^n)}\right), \qquad (1)$$
-
$$\Lambda_{K,L}^{n+1}(a,b) = F(\frac{a+b}{2}, \frac{a+b}{2}, Q_{K,L}^{n+1}, G_{K,L}) - F(a, b, Q_{K,L}^{n+1}, G_{K,L}).$$

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Stability and Existence

Stability of the saturation calculation

$$\boxed{0 \le s_K^n \le 1}$$

under (1) and the CFL condition

$$\delta t \le \inf_{K \in \mathcal{T}} \left(\frac{\phi_K m(K)}{\sum_{L \in N(K)} \left(2C_\eta(|Q_{K,L}^{n+1}| + |G_{K,L}|) + \Upsilon_{K|L} L_\varphi \right)} \right). \qquad (2)$$

Existence of solutions to the discrete system

An L^2-pressure estimate and a topological degree argument give the existence of solutions to the discrete system in pressure and saturation.

Convergence

Under assumptions on $\overrightarrow{Q}$, δt, $\theta_{K|L}^{n+1}$ and the sequence of the discretizations $(\mathcal{M}_m)_{m\in\mathbb{N}}$ we have

1. $s_m \xrightarrow{L^2(\Omega\times(0,T))} s \in L^\infty(\Omega\times(0,T)),$

2. $\varphi(s) \in L^2((0,T),H^1(\Omega)),$

3. $$\forall \psi \in C_{test}, \int_0^T \int_\Omega s\psi_t + f(s,\overrightarrow{Q},\overrightarrow{G}).\overrightarrow{\nabla}\psi - \overrightarrow{\nabla}\varphi(s)\overrightarrow{\nabla}\psi \; dxdt + \int_\Omega s_{ini}\psi(.,0)dx = 0$$

where $C_{test} = \{h \in H^1(\Omega\times(0,T)) \;/\; h(.,T) = 0\}$.

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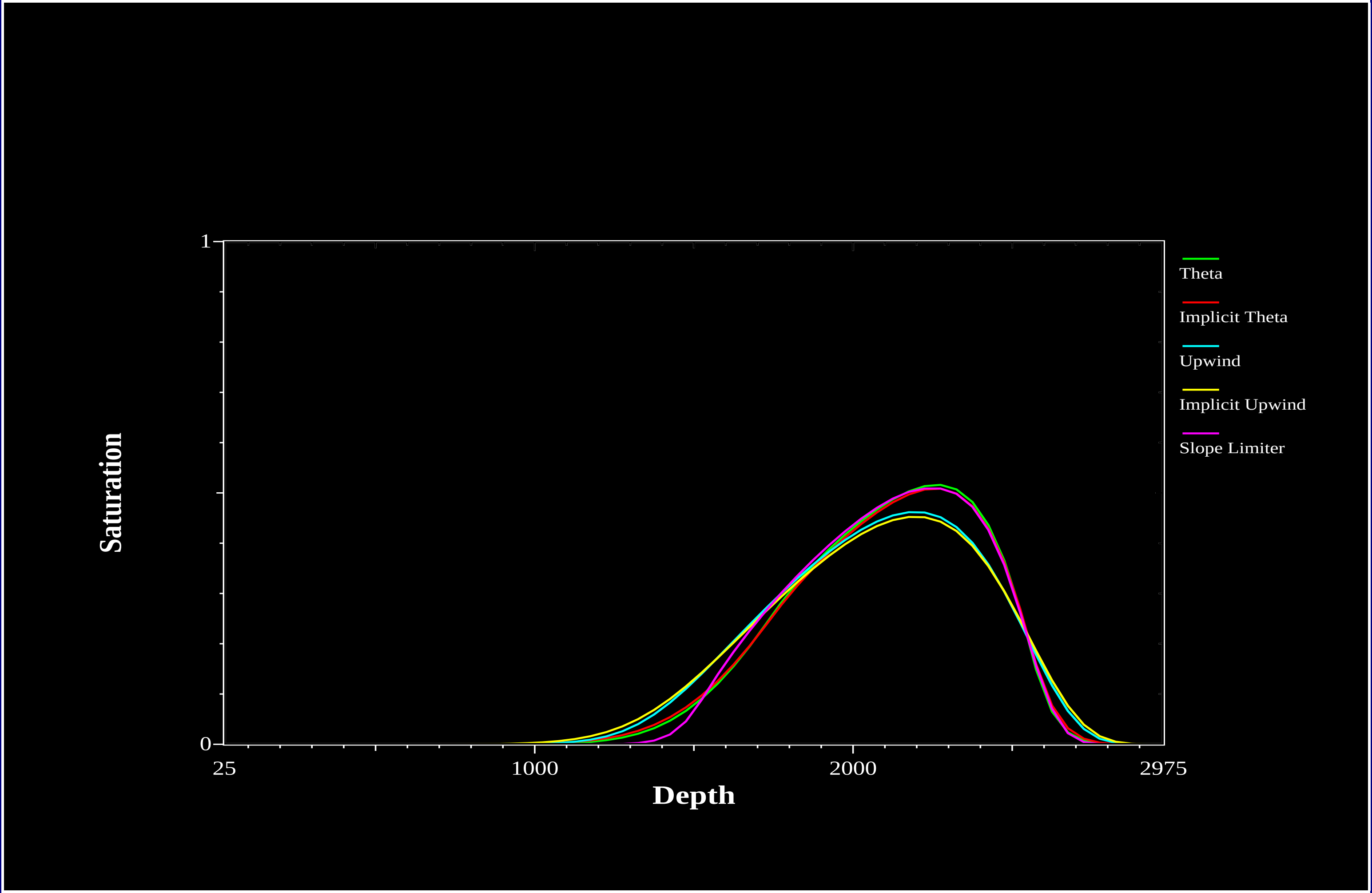
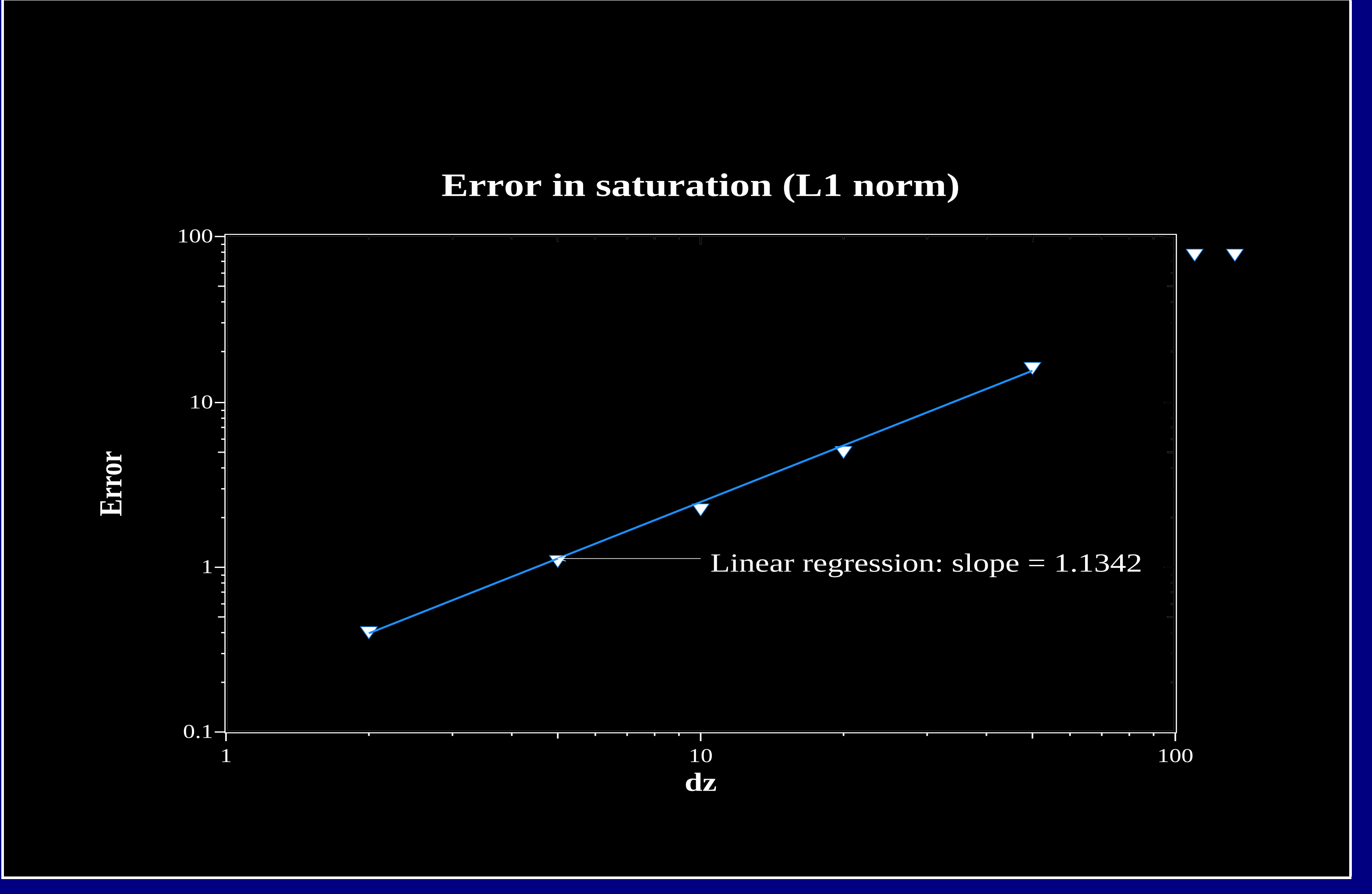
1D-test


1
0
25
1000
2000
2975
Depth
Saturation
Theta
Implicit Theta
Upwind
Implicit Upwind
Slope Limiter

Numerical convergence : Error in saturation



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Properties of the scheme :

- cheap and accurate,
- use for multidimensional domains,
- the implicit form is simple to implement.